ODE: Practice problems—Assessing methods

For the following equations, assess suitability of the basic three methods for solving them. Justify your opinion. You do not have to solve the equations.

1. $y' = 23y - 13.$

2. $y'' - y = \sqrt{x}.$

3. $y'y = x.$

4. $y' = xy + x.$

5. $y' = y^2 + x.$

6. $y' = x + y.$

7. $y'' = xy.$

Solutions

1. Separation: Suitable. Justification: Can be separated, $y' = 1\cdot(23y-13) = g(x)\cdot h(y)$, or $\int\frac{dy}{23y-13} = \int 1dx$.
Linear plus guess (undetermined coefficients): Suitable. Justification: The equation $y' - 23\cdot y = -13$ is linear with constant coefficients, the right-hand side -13 is special (we can guess for this type).
Linear plus variation: Perhaps suitable. Justification: It is linear with constant coefficients (or: is linear of order 1, each is enough). We cannot judge whether the integral for $C(x)$ is manageable unless we actually solve the problem.

2. Separation: Not suitable. Justification: It is not an equation od order 1.
Linear plus guess (undetermined coefficients): Not suitable. Justification: The equation $y''-1\cdot y = \sqrt{x}$ is linear with constant coefficients, but the right-hand side $\sqrt{x}$ is not special (we cannot make a guess for it).
Linear plus variation: Perhaps suitable. Justification: It is linear with constant coefficients. We cannot judge whether the integral for $C(x)$ is manageable unless we actually solve the problem.

3. Separation: Suitable. Justification: Can be separated, $y' = x\cdot\frac{1}{y} = g(x)\cdot h(y)$, or $\int y\,dy = \int x\,dx$.
Linear plus guess (undetermined coefficients): Not suitable. Justification: The equation is not linear.
Linear plus variation: Not suitable. Justification: The equation is not linear.

4. Separation: Suitable. Justification: Can be separated, $y' = x\cdot(1+y) = g(x)\cdot h(y)$, or $\int\frac{dy}{y+1} = \int x\,dx$.
Linear plus guess (undetermined coefficients): Not suitable. Justification: The equation $y' - x\cdot y = x$ is linear, but it does not have constant coefficients. (So we cannot find homogeneous solution.) By the way, the right-hand side x is special, but the guessing approach does not work without constant coefficients either.
Linear plus variation: Perhaps suitable. Justification: It is linear, and while it does not have constant coefficients, it is of order 1, so we can find y_h using separation. We cannot judge whether the integral for $C(x)$ is manageable unless we actually solve the problem.

5. Separation: Not suitable. Justification: Cannot be separated, y^2+x cannot be written as $g(x)\cdot h(y)$.
Linear plus guess (undetermined coefficients): Not suitable. Justification: The equation is not linear.
Linear plus variation: Not suitable. Justification: The equation is not linear.

6. Separation: Not suitable. Justification: Cannot be separated, y^2+x cannot be written as $g(x)\cdot h(y)$.
Linear plus guess (undetermined coefficients): Suitable. Justification: The equation $y' - 1\cdot y = x$ is linear with constant coefficients, the right-hand side x is special (we can guess for this type).
Linear plus variation: Perhaps suitable. Justification: It is linear with constant coefficients (or: is linear of order 1, each is enough). We cannot jusge whether the integral for $C(x)$ is manageable unless we actually solve the problem.

7. Separation: Not suitable. Justification: It is not an equation od order 1. We actually have a product $g(x)\cdot h(y)$ on the right, but it is of no use.
Since the equation $y'' - x\cdot y = 0$ is linear and homogeneous, we do not have to find particular solution y_p, so we need not worry about guessing (undetermined coefficients) or variation, we simply inquire about solving the homogeneous equation. Since it does not have constant coefficients, we cannot use the characteristic number approach and thus this approach is unsuitable.