# DEN: lab #9 (coronaversion)

Today we practice the alternative approach to solving equations via fixed point iteration.

1. In the previous worksheet we were looking for the root of the function $f(x) = x^2 - 3x + 1$ using the bisection method and the Newton method, with the latter we used the initial guess $x_0 = 1$.

   a) The standard transform of the equation $x^2 - 3x + 1 = 0$ to an equation of fixed point type is done by adding $x$. The resulting fixed point problem $x^2 - 2x + 1 = x$ has the function $\varphi(x) = x^2 - 2x + 1$. Write which iteration is used to find its fixed point, and evaluate the first three steps, assuming that our initial guess is again $x_0 = 1$.

   b) The magnitude $|\varphi'(x)|$ at the location of iteration provides us with a hint about the chances of success of this iteration. Here we would probably look at $\varphi'(x_0)$.
   Find this derivative. What is your guess regarding the chances of our iteration based on this derivative?

   c) Transform the equation $x^2 - 3x + 1 = 0$ to a fixed point problem, but in a different way than the standard approach from part a). Determine the iterating function $\varphi$, write the appropriate iterative formula, and calculate the first two steps of the iteration, that is, we want to see the first three approximations including the initial guess $x_0 = 1$.
   Find $|\varphi'(1)|$ and guess whether this iteration looks hopeful.

   d) Repeat the part c) including the test of $|\varphi'(1)|$, but with still another transformation into a fixed point problem.

2. In the previous worksheet, in the second problem we were solving the equation $x - 3 = \frac{1}{x^2}$. This is no longer a root-type problem, so there is no standard way to transform it into a fixed point problem—it is up to us.

   a,b) Find two distinct transformations of the given equation into a fixed point problem. For each of them, write the resulting iteration and show one step, that is, show the first two approximations including the initial guess $x_0 = 1$.
   Estimate the chances of success of your iterations based on $|\varphi'(1)|$.

   c) (bonus) We use relaxation to improve the convergence rate of our iterations.
   Choose one of the two transformations that you created above and apply the relaxation with parameter $\lambda = \frac{2}{3}$ to it. Write the appropriate iterative formula and show one step with the inital guess $x_0 = 1$.

   d) (bonus$^2$) For the chosen approach from part c), write the relaxed iterative formula with a general $\lambda$. Use $\varphi'_\lambda(x)$ to identify the optimal value of $\lambda$ the neighborhood of $x_0 = 1$.
   In fact, iterations hopefully quickly move to the root, which is near the number 3. Try to find the optimal value of $\lambda$ for a neighborhood of $x_0 = 3$.