DEN: solutions for lab #13 (coronaversion)

1. a) Separable? No.

Linear with guessing for RHS? $y' - 2y = 13e^{2x}$, yes.

Linear with variation? $y' - 2y = e^{2x}$ the type fits, it may work.

b) Separable? No.

Linear with guessing for RHS? No, y^2 is not a linear expression of the function y.

Linear with variation? No, y^2 is not a linear expression of the function y.

c) Separable? Yes, after a small modification: $y' = \frac{y-1}{x}$.

Linear with guessing for RHS? No. $y' = \frac{1}{x}y - \frac{1}{x}$ is linear, but the guessing method requires constant coefficients; moreover, we cannot guess the shape of solution for $b(x) = -\frac{1}{x}$.

Linear with variation? $y' - \frac{1}{x}y = -\frac{1}{x}$ the type fits, it may work.

(It turns out that we can't handle the integrals that appear, but the reasoning was sound.)

d) Separable? Definitely no, the ODE is not of order 1.

Linear with guessing for RHS? No. It is linear with constant coefficients, but we cannot guess the shape of solution for $\arctan(x)$.

Linear with variation? The type is $y'' - 3y = \arctan(x)$. This could work.

(Again, we run afoul of integrals when we try it.)

2. a) No. The set is linearly independent, but it has size two, while the space of solutions is three-dimensional.

b) The set is linearly independent and has the right number of functions, so it could be a fundamental system.

We can create e^x as a solution from $\lambda = 0$, solution 1 comes from $\lambda = 0$ and $x - 1$ can be obtained using correction, so we want the zero to be a double characteristic number. Remark: This will then create x, and using 1 and x we create $x - 1$ as a replacement for the basis according to one theorem from linear algebra.

So we want the characteristic polynomial $(\lambda - 1)(\lambda - 0)^2 = \lambda^3 - \lambda^2$. The given set of functions is a fundamental system of the equation

$$y''' - y'' = 0.$$

c) No. The set is not linearly independent, as $x - 1 = 1 \cdot (x + 1) - \frac{1}{3} \cdot 6$.

3. For $y = 1$ we have a stationary solution $y(x) = 1$ for $x \neq 0$.

For $y \neq 1$ we have $\int \frac{dy}{y-1} = \int \frac{dx}{x}$, $\ln|y - 1| = \ln|x| + c$, the familiar trick leads to $y - 1 = \pm e^c x$, hence $y(x) = 1 + Dx$, $x \neq 0$.

The stationary solution $y(x) = 1$ is included with the choice $D = 0$,

so the formula $y(x) = 1 + Dx$, $x \neq 0$ is a general solution.

4. a) y_h: $\lambda - 2 = 0$, $\lambda = 2$, $y_h = a\,e^{2x}$.

b) y_p: The expression $13e^{2x}$ leads to the guess $A\,e^{2x}$, but it has $\lambda = 2$, a match so correction is needed. We therefore want $y_p = Ax\,e^{2x}$.

We substitute into the equation: $A\,e^{2x} + Ax \cdot 2e^{2x} - 2Ax\,e^{2x} = 13e^{2x}$, $A = 13$, $y_p = 13x\,e^{2x}$.

Conclusion: The solution is $y(x) = 13x\,e^{2x} + a\,e^{2x}$, $x \in I\!R$.

5. a) y_h: $y' = \frac{y}{x}$, we solve by separation: $y(x) = 0$ is a stationary solution.

For $y \neq 0$ we get $\int \frac{dy}{y} = \int \frac{dx}{x}$, $\ln|y| = \ln|x| + c$, trick $y = \pm e^c x$, hence $y_h(x) = Dx$, $x \neq 0$.

b) y_p: Variation, $y_p = D(x)x$.

We substitute into the equation: $D'(x)x + D(x) = \frac{1}{x}D(x)x - \frac{1}{x}$, $D'(x)x = -\frac{1}{x}$.

Then $D'(x) = -\frac{1}{x^2}$, $D(x) = \frac{1}{x}$ (without $+C$ this time), thus $y_p = \frac{1}{x} \cdot x = 1$.

A general solution is $y(x) = y_p + y_h = 1 + Dx$, $x \neq 0$.

It is also possible to find $D(x) = 1 + C$, after substituting into the formula for y_p we get the general solution right away.