Laplaceova transformace

Základní obrazy:

$$e^{at} \triangleq \frac{1}{p-a}\,, \quad p > a \qquad (a \in \mathbb{R})$$

$$\cos\omega t \triangleq \frac{p}{p^2+\omega^2}\,, \quad p > 0 \qquad (\omega \in \mathbb{R})$$

$$\sin\omega t \triangleq \frac{\omega}{p^2+\omega^2}\,, \quad p > 0 \qquad (\omega \in \mathbb{R})$$

Další obrazy:

$$1 \triangleq \frac{1}{p}\,, \quad p > 0$$

$$t \triangleq \frac{1}{p^2}\,, \quad p > 0$$

$$t^2 \triangleq \frac{2}{p^3}\,, \quad p > 0$$

$$\vdots$$

$$t^n \triangleq \frac{n!}{p^{n+1}}\,, \quad p > 0 \qquad (n \in \mathbb{N})$$

Přepisy pro vzory:

$$\frac{1}{(p-a)^n} \triangleq e^{at}\,\frac{t^{n-1}}{(n-1)!} \qquad (a \in \mathbb{R},\ n \in \mathbb{N})$$

$$\frac{p-a}{(p-a)^2+\omega^2} \triangleq e^{at}\cos\omega t \qquad (a, \omega \in \mathbb{R})$$

$$\frac{\omega}{(p-a)^2+\omega^2} \triangleq e^{at}\sin\omega t \qquad (a, \omega \in \mathbb{R})$$

Základní vzorce:

$$a \cdot f(t) + b \cdot g(t) \triangleq a \cdot F(p) + b \cdot G(p)\,, \quad (p > \max\{p_f, p_g\}) \qquad \text{(linearita)}$$

$$t \cdot f(t) \triangleq -F'(p)\,, \quad p > p_f \qquad \text{(derivace obrazu)}$$

$$\frac{f(t)}{t} \triangleq \int_p^{+\infty} F(q)\,\mathrm{d}q\,, \quad p > p_f \qquad \text{(integrál obrazu)}$$

$$e^{at} \cdot f(t) \triangleq F(p-a)\,, \quad p > p_f + a \qquad \text{(posun v obrazu)}$$

$$f(at) \triangleq \tfrac{1}{a}\,F(\tfrac{p}{a})\,, \quad p > a \cdot p_f \qquad (a > 0,\ \text{změna měřítka})$$

$$f'(t) \triangleq p \cdot F(p) - f(0+)\,, \quad (p > \max\{0, p_{f'}\}) \qquad \text{(obraz derivace)}$$

$$\int_0^t f(u)\,\mathrm{d}u \triangleq \frac{F(p)}{p}\,, \quad (p > \max\{0, p_f\}) \qquad \text{(obraz integrálu)}$$

$$\int_0^t f(u) \cdot g(t-u)\,\mathrm{d}u \triangleq F(p) \cdot G(p)\,, \quad (p > \max\{p_f, p_g\}) \qquad \text{(obraz konvoluce)}$$

$$f(t) \cdot \mathrm{H}(t-a) \triangleq e^{-ap}\,\mathscr{L}\{f(t+a)\}\,, \quad p > p_f \qquad (a \geq 0,\ \text{translace})$$

$$f(t-a) \cdot \mathrm{H}(t-a) \triangleq e^{-ap}\,F(p)\,, \quad p > p_f \qquad (a \geq 0,\ \text{translace})$$

$$f(t) \triangleq \frac{F_T(p)}{1-e^{-Tp}}\,, \quad (p > 0) \qquad \text{(obraz periodické funkce)}$$

Další vzorce:

$$f''(t) \triangleq p^2\,F(p) - p\,f(0+) - f'(0+)$$

$$\vdots$$

$$f^{(n)}(t) \triangleq p^n\,F(p) - p^{n-1}\,f(0+) - \cdots - f^{(n-1)}(0+)$$