Syllabus: BE5B01LAL - Linear Algebra

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Text: P. Pták: Introduction to Linear Algebra. CVUT, Praha, 1997 or 2005.
ftp://math.feld.cvut.cz/pub/krajnik/vyuka/ua/linalgeb.pdf

Course description: This is a standard introductory course of linear algebra. It focuses on the related concepts of linear space and linear transformations (linear independence, bases and coordinates) and matrices (determinants, inverse matrices, linear mappings, eigenvalues). As an applications we study the geometry of 3-dimensional space (including scalar and vector product, orthogonality, equations of lines and planes). Bilinear and quadratic forms are introduced.

Lectures Attendance is not obligatory but highly recommended. Students are encouraged to study material covered in the lectures before attending the lab.

Labs are devoted to develop the necessary technical skills for problem solving. During the labs of Week 6 and Week 11 a test (45 min., 3 questions for a total of 20 points) will also be handed out. **Attendance is obligatory**: In order to obtain the certificate of attendance (needed for the final exam), students are required to actively participate in the laboratory class, hand in the assigned homework and obtain a sufficient score (at least 9 points out of 20) during lab tests. If the score obtained in a test is not sufficient, extra homework will be assigned and the test will be repeated at the end of the course.

Exam. Students who obtain attendance certificate (”zapocet”) are allowed to take the exam. The exam is composed of written and oral part. The written final exam will be in January-February, exact dates will be announced later, it will consist of several problems to be solved in 90 minutes for a total of 90 points. The oral final exam is optional, it is used to improve the grade up to ten points. Questions about theory will be asked (definitions, theorems, proofs). In order to pass the exam a minimum of 50 points in the written test is required, students with more then 60 points in the written part of the exam will be allowed to improve their grade with the oral part of the exam.

Grades are assigned as follows: F(<49pts), E(50-59), D(60-69), C(70-79), B(80-89), A(90-100).

Content of lectures.

(week 1) Polynomials. Introduction to systems of linear equations and Gauss elimination method.
(week 2) Linear spaces, linear dependence and independence.
(week 3) Basis, dimension, coordinates of vectors.
(week 4) Matrices: operations, rank, transpose.
(week 5) Determinant and inverse of a matrix.
(week 6) Structure of solutions of systems of linear equations. Frobenius Theorem.
(week 7) Linear mappings. Matrix of a linear mapping.
(week 8) Eigenvalues and eigenvectors of matrices and linear mappings.
(week 9) Similarity of matrices, matrices similar to diagonal matrices.
(week 10) Free vectors. Dot product and cross product.
(week 11) Euclidean space, orthogonalization, orthonormal basis. Fourier basis.
(week 12) Lines and planes in 3-dimensional real space.
(week 13) Introduction to bilinear and quadratic forms.

Detailed course description:

Polynomials: definition and operations, Horner's rule, roots of polynomials, Fundamental Theorem of Algebra and its consequences. Linear equations. Introduction to Gauss elimination method to solve systems of linear equations. Axiomatic definition of linear space and examples: real numbers, matrices, vectors in R^n, polynomials, real functions. Subspaces. Linear combination, linear dependence and independence of vectors of a linear space. Span, set of generators, basis and dimension of a linear space. Finite dimensional linear spaces, coordinates of vectors, transition matrix from a basis to a new one.

Matrices: multiplication and its properties. Definition and computation of rank, transpose, determinant, inverse of a matrix. Elementary row operations, echelon and canonical form of a matrix. Linear dependence and independence of rows, regularity of a matrix. Cramer's rule for solving n-times-n systems of linear equations. Structure of the solution set of homogeneous and not homogeneous systems of linear equations, Frobenius Theorem.

Linear transformations: definition and examples. Image and Kernel of a linear map, their basis and dimensions. For linear transformations between finite dimensional spaces: Theorem on dimensions of Kernel and Image, linear transformation defined on vectors of a basis, matrices associated to a linear transformation with respect to different basis. Similarity of matrices, matrices similar to a diagonal matrix. Eigenvalue and eigenvectors of a matrix and a linear map, diagonalization of a matrix. Composition of linear maps, existence and evaluation of inverse of a linear map.

3D real vectors, definition and properties of scalar product, cross product, norm and distance. Euclidian spaces, Cauchy-Schwarz inequality, angle between non-zero vectors, orthogonality of vectors, projections, Fourier coefficients. Gram-Schmidt orthogonalization process, orthonormal basis of a linear space. Equation of line and planes in 3D real space, distance of a point from a straight line. Introduction to bilinear and quadratic forms, conics and associated matrices.