Exercise: Draw a Hasse diagram for \((A,\mid)\) (divisibility relation), where

(i) \(A = \{1, 2, 3, 4, 5, 6, 7, 8\}\);  
(ii) \(A = \{1, 2, 3, 5, 11, 13\}\);  
(iii) \(A = \{2, 3, 4, 5, 6, 30, 60\}\);  
(iv) \(A = \{1, 2, 3, 6, 12, 24\}\);  
(v) \(A = \{1, 2, 4, 8, 16, 32, 64\}\);  
(vi) \(A = \{2, 4, 6, 12, 24, 36\}\).

Exercise: Consider the poset \((\{3, 5, 9, 15, 24, 45\}, \mid)\), that is, the divisibility relation.

(i) Draw its Hasse diagram.
(ii) Find its maxima, minima, greatest and least elements when they exist.
(iii) Find maxima, minima, greatest and least elements of the set \(M = \{3, 9, 15\}\), when they exist.

Solution:

(i): \[
\begin{array}{c}
\text{8} \\
\text{4} & \text{6} \\
\text{2} & \text{3} & \text{5} & \text{7} \\
\end{array}
\]

(ii): \[
\begin{array}{c}
\text{2} & \text{3} & \text{5} & \text{11} & \text{13} \\
\end{array}
\]

(iii): \[
\begin{array}{c}
\text{60} \\
\text{4} & \text{6} \\
\text{2} & \text{3} & \text{5} \\
\end{array}
\]

(iv): \[
\begin{array}{c}
\text{24} \\
\text{12} \\
\text{6} \\
\text{2} & \text{3} & \text{1} \\
\end{array}
\]

(v): \[
\begin{array}{c}
\text{16} \\
\text{4} & \text{8} \\
\text{2} & \text{1} \\
\end{array}
\]

(vi): \[
\begin{array}{c}
\text{36} \\
\text{24} \\
\text{2} & \text{3} & \text{6} \\
\end{array}
\]

Solution:

(ii): Max 24,45, greatest DNE, min 3,5, least DNE.
(iii): Max 9,15, greatest DNE, min 3, least 3.

\[
\begin{array}{c}
\text{45} \\
\text{24} \\
\text{9} & \text{15} \\
\text{3} & \text{5} \\
\end{array}
\]