Dokonalá čísla, zvláště to páté [In Czech]

Jindřich Bečvář
Jindrich.Becvar@mff.cuni.cz

Abstrakt
River-crossing problems or from the pun to the strategy

Martina Bečvářová

becvamar@fd.cvut.cz

Abstract

In our presentation, we explain one old mathematical problem - river-crossing problems. We shortly indicate mathematical methods which helped their solving, i.e. the old method of "trial and error", methods of using tables, graph theory and linear programming. We describe the historical sources originated in European Middle Ages (the 8th century), European Renaissance (the 15th century) and the modern time (the 17th century).

We will start with Alcuin of York's famous mathematical collection Propositiones ad acuendos iuvenes and we will analyse his examples nr. 17, 18, 19, 20. The original essence of the examples lies in finding the right one, and if possible, the optimal approach crossing of the river in the boat, if we have a limited transmission capacity and the predetermined conditions to be complied with (the total mass of freight, some risks of damage to the boat, some social conventions, etc.). Alcuin is formulated them as the cheerful and amusing puns that are often unrealistic (i.e. rowing animals) and non-upbringing (rowing women or children and men watching from the other shore and inventing transport conditions, etc.). However, the problems allow refinement of logical thinking and creating strategies, i.e. finding an efficient algorithm that would be universally usable and understandable. Alcuin's solutions are expressed verbally (step by step he described the situation arising from the banks) but their optimality is not even discussed.


2Anglo-Saxon monk Alcuin of York (about 735–804) is considered to be the founder of the classical river-crossing problems. In his youth, he studied at the cathedral school in York, where he later worked as a teacher and an administrator of the famous library. He soon became a renowned scholar and one of the best scholars of Western Europe. In 781 in Parma, he met Charlemagne (about 747–814), who invited him to his court and asked him to reorganized Frankish education system. Alcuin accepted the offer and for more than a decade working in the Frankish empire. Around 796, he applied for exemption from Charlemagne’s services and became the abbot of the monastery of St. Martin of Tours in France.
The 17th example: They were three brothers, each of whom had a sister and they wanted to transfer across the river. Each of them felt a longing for his friends’ sisters. When they came to the river, they found a small boat in which they could not carry more than two of them simultaneously. Tell me, who can you, as they can across the river without one of them was tainted. ([2], p. 11)

The 18th example: A man wanted to transfer across the river the wolf, the goat and the cabbage and he could not find another boat than the one that was capable of carrying only two of them. But, he wanted that all has transferred completely undamaged. Tell me, if you can how he could organize their transport. ([2], p. 11)

The 19th example: A man and a woman, each weighing one talent, have two children who together weigh one talent too. They wanted to transfer across the river. They found a small boat that cannot carry more than one talent. Let make a transport without the boat sank. ([2], p. 11)

The 20th example: Hedgehog and his partner having two children weighing a pound and they want to transfer across the river. ([2], p. 11)

We will continue with Luca Pacioli’s³ manuscript De viribus quantitatis (written between the years 1496 and 1508) which still is not widely known and deeply appreciated by historians of mathematics⁴. De viribus quantitatis is one of the first European works on recreational mathematics, "magical effects", "scientific experiments" and logical gambles. It is a source for refining the intellect, education and entertainment in which the foundations of mathematical thinking are unloaded by means of engaging and inspiring examples. L. Pacioli stated not only texts of problems and their solutions, but he gave also instructions on how to create and witty puns some logical games and tasks to an audience. His examples include the popular "stories" that have arouse interest in studying mathematics, demystify tricks, puzzles and puns, and in particular show the power of human reason and the beauty of mathematics. In the first chapter of the manuscript De viribus quantitatis, L. Pacioli presented four river-crossing problems (examples nr. 49, 50, 51, 52) which are now known as the so called the jeep problem.⁵

The 49th example: One citizen from Borgo has 90 apples, which are to be delivered to his friend to Perugia (30 miles away). He hired a carrier that can carry only 30 apples and as cargo, consuming 1 apple for 1 mile. How can I do to fetch maximum of apples in Perugia? ([3], p. 42)

³Luca Pacioli (about 1445–1515) was the famous Italian Renaissance mathematician, university professor, translator, author of the huge mathematical encyclopaedia and several books on mathematics, geometry, accounting, recreational mathematics and theory chess game.

⁴For more information see [3], the river-crossing problems are described at the pages 42, 43 and 48.

⁵The modern jeep problem is the most common modification of the classical river-crossing problem in which water, food or fuel are transported (resp. warehouse and pumping stations are built at the optimal distance from the bases). The transport takes place at the predefined transmission capacities and the condition that assumes that the commodity is consumed in accordance with the predetermined conditions during the transport (usually zero consumption even under zero loads). Current examples assume a more realistic scenario, i.e. the consumption depends on the amount of transported goods, but it is never zero. The aim of the task is, of course, the optimization of distributions, respectively stockpiling.
L. Pacioli described two different alternatives, the second one is less than optimal, but optimization is neither stressed nor proved. Another Pacioli’s examples are just modifications of the original conditions of carriage. At the end of the jeep problem, he advised readers to choose a better carrier or realistic conditions of carriage or to study mathematics.

In the first chapter of the manuscript Pacioli also presented one classical river-crossing problems (the 61st example): There were three men and each of whom has a woman. They had to across the water. The men were very jealous. When they came to the water, they found a small boat in which they could not carry more than two of them simultaneously. How did they manage the transport across the water without a single woman was tarnished? ([3], p. 48)

In his witty comments, Pacioli recalled that residents of the famous town Venice and the less famous town Chioggia solve this problem every day and are able to deal with it very well. His task is "more realistic and fun" than Alcuin’s one. From a mathematical point of view, however, this is just a mere verbal modification of Alcuin’s old example because its nature and solution are absolutely identical.

The river-crossing problems were also described by Claude Gaspard Bachet de Méziriac, who in 1612 in Lyon published the popular book Problemes plaisans et delec-tables, qui se font par les nombres ... It contained various arithmetic and algebraic puns, and numerous card tricks, river-crossing problems, constructions of magic squares, Josephus problem, problems of weighing and pouring liquids etc. It is interesting, that almost no space is provided to the geometric puns and interesting geometric constructions. Bachet de Méziriac drew his inspiration from the works of Alcuin, Leonard of Pisa called Fibonacci (about 117–1250), Manuel Moschopoulos (the 13th/14th century) and Nicole Tartaglia (1499–1557).

Bachet’s example: One big troop of soldiers had to across the river, but the bridge was demolished and the ford did not exist. At the shore, two boys played in a small boat, which can carry a maximum of one adult or two children. How did a company with this boat could make the transport across the river? ([1], p. 137)

Bachet de Méziriac described only the first step of the solution. To transport of only one soldier it must be done four trips across the river (rowing one or both boys). To transport all troop (i.e. n soldiers) it is necessary to perform 4n paths. Such transport of the troop is completely unrealistic in terms of time and forces.

The reader may ask a provocative question: What good are river-crossing problems and knowledge of their solution? And he or she may answer: It is just a nice pun and fun. But it is not such insignificant task as it might seem at the first glance. It enables the development of logical and strategic thinking, thinking of algorithms and optimization

---

6Claude Gaspard Bachet de Méziriac (1581–1638) was a French mathematician, poet, linguist, writer, lover of arithmetic, an expert in ancient mathematics and ordinary member of the Paris Academy of Sciences. He became famous in 1621, when he published his own Latin translation of Diophantos’ Arithmetic [Diophanti Alexandrini arithmeticon libri sex ...] which was studied by French mathematician Pierre Fermat (1607/8–1665), who made the note on the edge of his specimens in which he formulated the famous Last Fermat Theorem.
solutions. Curiously, the US military uses the jeep problem by the task of testing the ability of strategic thinking and the ability to plan operations for candidates for leadership positions.

References


Gibbs-Voronoi tessellations in $\mathbb{R}^3$

Viktor Beneš²
benes@karlin.mff.cuni.cz

Abstract
This is a review talk about random tessellations in the space which can be used to the modeling of the grain network of polycrystalline materials. We discuss stationary Gibbs models where the geometry of grains is controlled by suitable energy functions. The approaches to the simulation of the tessellation in a bounded window are mentioned. The estimation of parameters is based on standard methods from spatial statistics. The quantification of the degree-of-fit between the model and data is discussed.

References
On persistence of convergence of kernel estimates in particle filter

David Coufal

coufal@karlin.mff.cuni.cz

Abstract
Persistence of convergence of kernel estimates in particle filtering over time is investigated. Convergence of kernel estimates is ensured when certain assumptions on theoretical filtering densities hold at any time of operation of the filter. These assumptions can be directly checked at the start of operation of the filter, but in the course of time they can be assessed only indirectly by investigating properties of transition kernels of the signal process. In the contribution, we show that the convergence assumptions are assured when the conditional characteristic function of the transition kernels is bounded by the characteristic function of a common Sobolev function. The result complements the result on convergence of kernel estimates so that it applies at any time instant of operation of the filter and presents a handy tool for checking this desirable property.

Acknowledgment: This work was supported by grant SVV-2016-260334 of Charles University in Prague.
Abstract
In this talk, we discuss space-time regularity of solutions to linear SPDEs with additive Volterra noise. We consider a class of stochastic processes which might not be Gaussian or semimartingales but which admit a certain covariance structure instead. The most notable examples are the fractional Brownian motion of $H > 1/2$ (Gaussian) and the Rosenblatt process (non-Gaussian). Theory of stochastic integration with respect to Volterra processes in $L^p$ spaces is developed and sufficient conditions for (time) continuity of a mild solution to a stochastic evolution equation with values in $L^p$ are given. In particular cases of SPDEs (e.g. the stochastic heat equation), this allows to prove continuity of the solution in the space variable as well.
On efficient evaluation of certain integrals appearing in the moment estimation methods for point processes

Jiří Dvořák

dvorak@karlin.mff.cuni.cz, dvorak@utia.cas.cz

Abstract
In the classical moment-based parametric inference for point processes the popular methods are, among others, the composite likelihood and the Palm likelihood. However, rather complicated integrals appear in the respective criteria to be optimized. Such integrals need to be evaluated in each step of the numerical optimization procedure and hence it is beneficial to have a fast way for their computation.

We present a rather general and efficient way of numerical evaluation of the integrals of the type

\[ \int_W \int_W \lambda(u)\lambda(v)g(u - v)\mathbb{I}(\|u - v\| \leq R) \, du \, dv. \]

In the context of composite likelihood method for point processes \( W \) is the observation window, \( \lambda \) is the first-order intensity function, \( g \) is the pair-correlation function (assumed to be translation-invariant), \( \mathbb{I} \) is the indicator function and \( R > 0 \) is a user-specified tuning constant.
Testing equality in distribution of random convex compact sets via theory of random hedgehogs and $\mathcal{N}$-distances

Vesna Gotovac
vgotovac@pmfst.hr

Coauthor: Kateřina Helisová

Abstract
The contribution presents the results from [1]. The main idea is the following. The space of convex compact subsets is equipped with Minkowski addition and multiplication by non negative real numbers and these operations correspond to operations on their support functions. However, it does not form a vector space. Using the vector space of hedgehogs, which are defined as differences of convex compact sets, the definition of random convex compact set can be extended to the definition of random hedgehog. Moreover, their characteristic functions can be defined. Using these terms and the theory of $\mathcal{N}$-distances introduced in [2], we can derive a statistical test for testing equality in distribution of two random hedgehogs.

References


Abstract
The first idea of logarithms appeared in the treatise [4] of John Napier. Three years later H. Briggs published the first table [1] of common logarithms (of base 10) and J. Napier then construed his own table [3].
Today we calculate logarithms in the first year course on calculus using Taylor series. Taking into account that differential calculus appeared five decades after the publication of the table [1], we can ask how the table was generated.
The main aim of the talk is to show how can the common logarithms be calculated without any use of differential calculus and why the natural logarithms are “better” (and in which sense) than the common logarithms.
In the second part we will focus on the question how the ancient problem of quadrature of a rectangular hyperbola (traced back to Archimedes (?), compare his treatise The Quadrature of the Parabola) was successfully solved in the middle of the 17th century by Gregorius a Sancto Vincentio in [2].

References
Assessing dissimilarity of random sets through convex compact approximations, support functions and envelope tests

Kateřina Helisová
helisova@math.feld.cvut.cz
Coauthors: Vesna Gotovac, Ivo Ugrina

Abstract
The contribution presents the results from [1]. It concerns a measure of dissimilarity of stationary and isotropic random sets through a heuristic based on convex compact approximations, support functions and envelope tests introduced in [2]. The measure is used to distinguish between two realisations of random sets, more precisely to decide whether two given realisations come from the same underlying process when we have their pixel images. The suggested procedure is justified through simulation studies of common random models like Boolean and Quermass-interaction processes with different parameters.

Acknowledgment: Supported by The Czech Science Foundation (GAČR), project No.13-05466P.

References


Gradual changes in two-sample setup

Zdeněk Hlávka
hlavka@karlin.mff.cuni.cz

Coauthor: Marie Hušková

Abstract
Motivated by gender and age specific measurements, we formulate and investigate the so-called two-sample gradual-change hypothesis. In homoscedastic setup, we derive asymptotic distribution of the least squares change-point estimator. In heteroscedastic situation, we use bootstrap approximation to test hypotheses concerning change-point location and to construct corresponding confidence intervals. Interestingly, the change-point approach is more powerful than standard two-sample t-tests.
Intertwining of the Wright-Fisher diffusion

Tobiáš Hudec

tobias.hudec@gmail.com

Abstract
It is known that the time until a birth and death process reaches a certain level is distributed as a sum of independent exponential random variables. Diaconis and Miclo [1] and Swart [2] gave a probabilistic proof of this fact by coupling the birth and death process with a pure birth process such that the two processes reach the given level at the same time. Their coupling is of a special type called intertwining of Markov processes. We apply this technique to couple the Wright-Fisher diffusion with reflection at 0 and a pure birth process. We show that in our coupling the time of absorption of the diffusion is a.s. equal to the time of explosion of the pure birth process. The coupling also allows us to interpret the diffusion as being initially reluctant to get absorbed, but later getting more and more compelled to get absorbed.

Acknowledgement: This talk is based on my master thesis. I would like to thank my thesis supervisor Dr. Jan M. Swart for his advice and help.

References


Estimating the conductivity tensor in mouse embryonic heart

Jiří Janáček\textsuperscript{12}

jiri.janacek@fgu.cas.cz

Coauthor: David Sedmera\textsuperscript{12}

Abstract
His–Purkinje System in embryonic hearts was visualized in 3D using confocal microscope [1]. The images were segmented and skeletonized. The conductivity tensor of HPS was estimated using the heat kernel of the skeleton.

Acknowledgement: This work was supported by the Czech Science Foundation, grant 13-12412S.

References

Parameter estimation for the stochastic differential equation of second order

Josef Janák²
janak@karlin.mff.cuni.cz

Abstract
Consider the following wave equation
\[ \frac{\partial^2 u}{\partial t^2}(t, \xi) = b\Delta u(t, \xi) - 2a \frac{\partial u}{\partial t}(t, \xi) + Q^{1/2} \dot{B}(t, \xi), \quad (t, \xi) \in \mathbb{R}_+ \times D, \]
\[ u(0, \xi) = u_1(\xi), \quad \xi \in D, \]
\[ \frac{\partial u}{\partial t}(0, \xi) = u_2(\xi), \quad \xi \in D, \]
\[ u(t, \xi) = 0, \quad (t, \xi) \in \mathbb{R}_+ \times \partial D, \]
where \( D \subset \mathbb{R}^d \) is a bounded domain with a smooth boundary, \( a > 0, b > 0 \) are unknown parameters, \( Q \) is a positive nuclear operator on \( L^2(D) \) and the \( \dot{B}(t, \xi) \) is the formal time derivative of the Brownian motion.
Based on the observation of trajectory of process \( \{X_t = (u(t, \cdot), \frac{\partial u}{\partial t}(t, \cdot))^\top, 0 \leq t \leq T\} \), the strong consistent estimators of parameters \( a \) and \( b \) will be proposed.

Acknowledgement: This work was supported by the GAČR grant No. 15–08819S – Stochastic processes in Infinite Dimensional Spaces.

References
Maintenance optimization of a continuously deteriorating parallel system using simulated annealing

Čeněk Jirsák

cenek.jirsak@tul.cz

Abstract
Maintenance optimization is a common topic in mathematical reliability theory. Motivation for our model are continuously deteriorating systems consisting of components working in parallel with a redundancy. An example of such a system might be a group of coal mills in a power plant as in [1], where 7 out of 8 mills must be operating for an efficient coal burning. Such systems are usually modeled as multistate stochastic systems using standard tools (mainly Markov chains and processes) as in [1,2] and references therein.

Our focus is to apply continuous deterioration to the model. So far we can find an optimal policy only for a simple deterministic models. For more complex models we use numerical approximation such as simulated annealing.

Acknowledgment: The work was supported by the Ministry of Education of the Czech Republic within the SGS project on the Technical University of Liberec.

References


Abstract
In this contribution, controlled linear stochastic evolution equations driven by square
integrable Lévy processes are studied in the Hilbert space setting. The control operator
may be unbounded which makes the results obtained in the abstract setting applicable
to parabolic SPDEs with boundary or point control. The first part contains some prelim-
inary technical results, notably a version of Itô formula which is applicable to weak/mild
solutions of controlled equations. In the second part, the ergodic control problem is
solved: The feedback form of the optimal control and the formula for the optimal cost
are found. As examples, various parabolic type controlled SPDEs are studied.
On weakly dependent random fields

Jana Klicnarová
klicnarova@ef.jcu.cz

Abstract
The problem of central limit theorem and invariance principle for weakly dependent random fields based on martingale approximation has been solved from the eighties of the last century. Recently, there were a lot of new results on this topic published. Our aim is to overview these new results – to introduce weak limit theorems for stationary random fields.
Alternative K-functions for stationary point processes

Kateřina Koňasová
konasova.k@seznam.cz

Abstract
K-function has often been regarded as one of the most useful tools for the analysis of point patterns. Some modifications of this summary characteristic in planar case were developed in order to investigate for a possible anisotropy. We introduce so-called directional K-function and illustrate its major contribution in orientation analysis of point patterns. We introduce a heuristic method for detecting anisotropies in clustered or regular data.

References


EBSD in shape memory studies

Jaromír Kopeček
kopecek@fzu.cz

Abstract
The most relevant shape memory alloy is NiTi intermetallic compound called Nitinol. It already has many engineering applications, particularly in medicine. Thin superelastic NiTi wires are used to manufacture medical devices such as stents. Functional properties of NiTi wires depend on its grain microstructure which can be conveniently characterized by destructive 3D-EBSD method. We have a technology to precisely manipulate microstructure of thin metallic wires by thermomechanical processing using electropulse heating under controlled force. The grain microstructure of a commercial thin NiTi wire in cold worked state was intentionally manipulated by electropulse processing to create grains of 6 µm in diameter in order to investigate the phenomenon of localized tensile deformation of NiTi in martensite bands by 3D x-ray diffraction method [1]. The 3D-XRD method allows for in-situ nondestructive evaluation of grain microstructure. Hence we were able to compare the microstructure representation in NiTi obtained by 3D-XRD and 3D-EBSD methods. Synchrotron 3D-XRD provides information on large number of grains with less precision, whereas 3D-EBSD captures fewer grains but reveals real shapes of grains and grain boundaries.

References

Markov processes in reliability theory

Vít Kubelka²
kubelkavit@gmail.com

Abstract
In this talk, we deal with application of Markov processes in analysis of reliability of complex industrial systems. We describe a general algorithm whose input is a special form of a failure tree, which describes reliability of a certain system, and whose output is a Markov process, which describes evolution of reliability of the given system in time. Unlike the classical failure tree approach, Markov processes enable to consider dynamical evolution of the system and reparation of components. Using classical Markov processes theory we solve an important problem, which is crucial for strategic decisions concerning the management of nuclear power plant Temelín. It is not possible to solve this problem effectively using only failure trees.

References

Parameter estimation in SPDEs - asymptotic properties
Pavel Kříž
krizp@vscht.cz

Abstract
We shall present new results on asymptotic (in time) properties of the so-called ergodic estimator of the drift parameter in linear SPDEs driven by fractional Brownian motion. Strong consistency is shown using ergodicity and asymptotic normality using the celebrated 4th moment theorem (following from the Malliavin calculus and Stein method). We shall also discuss space asymptotics (increasing number of dimensions) and possible improvements of the estimator.

References
Analysis of renewable resources production Benefits of RES power plants aggregation

Michaela Lachmanová

Abstract

The increasing electricity production from RES is causing significant problems within the power grid. These problems result from the low predictability and high volatility of electricity production. The integration of renewables into the power grid causes among others the requirement for increased level of auxiliary and system services, or electricity market distortion. One of promising ways how to successfully integrate renewables is thorough analysis of RES dynamic behavior.

The paper deals with a complex analysis of the dynamic behavior of specific sources based on renewable energy sources, identification of dependencies between intermittent sources and evaluation of the RES aggregation effects. The main aim of this paper lies in evaluation of aggregation of sources and in optimization of structure of sources portfolio. This optimized portfolio can be used to create a portfolio of sources for electricity trading for minimizing the investor’s risk or knowledge of optimal structure of portfolio can be used for dimensioning of set of new resources to optimize grid operation.
LQ control for stochastic PDEs with Volterra noise

Bohdan Maslowski
maslow@karlin.mff.cuni.cz

Abstract
In the first part, an infinite-dimensional linear-quadratic control problem for equations with additive Gaussian fractional noise on finite and infinite time horizon is solved. The feedback form of the optimal control and the optimal cost are given explicitly. The optimal control is the sum of a suitable prediction of the system response to the future noise and the well known linear feedback control for the associated deterministic linear-quadratic control problem. The covariance of the noise as well as the control operator in the system equation may in general be unbounded, so the results can also be applied to the case of boundary/point noise and control of stochastic PDEs. In the second part, analogous control problems are studied for SPDEs with bilinear scalar Gaussian noise of Volterra type. These noise processes include fractional Brownian motions with the Hurst parameter $H \in (\frac{1}{2}, 1)$, Liouville fractional Brownian motions with $H \in (\frac{1}{2}, 1)$, and some multifractional Brownian motions. The family of admissible controls for the quadratic costs is a family of linear feedback controls. This restriction on the family of controls is made to facilitate the implementation of the optimal controls. The bilinear equations have drift terms that are linear evolution operators. These equations can model stochastic partial differential equations of parabolic and hyperbolic types.

References


Convergence of generalized entropy minimizers in sequences of convex problems

František Matúš
matus@utia.cas.cz
Coauthor: Imre Csiszár

Abstract
Let $(Z, \mathcal{Z}, \mu)$ be a $\sigma$-finite measure space. For real functions $g$ on $Z$, let

$$H_\beta : \ g \mapsto \int_Z \beta(z, g(z)) \mu(dz).$$

The functional $H_\beta$ is based on an integrand $\beta : Z \times \mathbb{R} \to (-\infty, +\infty]$. Assumptions on $\beta$ include strict convexity in the second coordinate, finiteness of $\beta(z, t)$ for $t > 0$ and $\beta(z, t) = +\infty$ for $t < 0$.

The integral functional $H_\beta$ is minimized over a convex set. A generalized minimizer exists under a boundedness condition. Sequences of the minimization problems are studied when the constraint sets are nested. The corresponding sequences of generalized minimizers are related to the minimization over limit convex sets. Martingale theorems and moment problems are discussed.

References


STIT tessellations – Markov processes of tessellations that are generated by cell division

Werner Nagel
werner.nagel@uni-jena.de

Abstract
We consider random processes where the states are tessellations of the plane or a higher-dimensional Euclidean space. The transition to a new state is caused by a random division of individual cells, where each cell has a random life time, and at the end of its life it is randomly divided. There are many different and flexible models for such processes. Among these models, the STIT tessellation (stochastically STable under the operation of ITeration of tessellations) appears to be the most interesting one from a theoretical point of view, and also a potential reference model for applications. It was introduced by W. Nagel and V. Weiß [1]. In the talk, a survey of important features is given, and also recent results on ergodic properties (joint work with S. Martínez, [2]).
In contrast to alternative models for fracture (or fissure or crack) systems which occur inside a material or on a coating of a surface (craquelée effect) STIT allows for theoretical investigations (and not only simulation studies) which reveal essential properties and relations for its parameters.

References


Investigation of microstructure-property relationships by a large simulation study based on random set models

Matthias Neumann
matthias.neumann@uni-ulm.de

Abstract
The analysis of big data is changing industries, businesses and research since large amounts of data are available nowadays. However, in the area of microstructures, acquisition of (3D tomographic image) data is difficult and time-consuming. It is shown that large amounts of data representing the geometry of virtual, but realistic 3D microstructures can be generated using stochastic microstructure modeling. Combining the model output with physical simulations and data mining techniques, microstructure-property relationships can be quantitatively characterized. Exemplarily, we aim to predict effective conductivity given the microstructure characteristics volume fraction $\epsilon$, mean geodesic tortuosity $\tau_{\text{geod}}$ and constrictivity $\beta$, which turned out to be meaningful for effective conductivity $\sigma_{\text{eff}}$ [3].

In the talk the mathematical definitions of $\tau_{\text{geod}}$ and $\beta$ for stationary random closed sets are given [1]. Moreover, we analyze the influence of these microstructure characteristics on $\sigma_{\text{eff}}$ based on 8119 virtual microstructures generated by the aid of two different 3D random set models. This is - to the best of our knowledge - by far the largest set of virtual microstructures that has ever been analyzed. Fitting artificial neural networks, random forests and classical equations, the prediction of effective conductivity based on geometric microstructure characteristics is possible [2].

References


Stein method for functionals of point processes with conditional intensity

Daniela Novotná

dnlnvt@gmail.com

Abstract

To quantify the discrepancy between distributions of two random variables $X_1, X_2$ we can use the Wasserstein distance. Consider a point process $\mu$ on a measurable space $(\mathcal{X}, \mathcal{X})$. Then using the Stein’s method we can under some additional assumptions find explicit bound for the Wasserstein distance between functional of $\mu$ and the standard normal distribution. This method was first demonstrated on functionals of Poisson process, cf. [1]. Recently in [2] the Stein’s method was applied to innovations of stationary point processes with Papangelou conditional intensity. As a special case, the central limit theorem induced by the Wasserstein distance can be shown for Gibbs point process with pair potential.

References


Limit theorems for random tessellations

Zbyněk Pawlas

pawlas@karlin.mff.cuni.cz

Abstract
The aim of this talk is to review some asymptotic results for random tessellations. The main focus is on stationary Poisson-Voronoi tessellations and Poisson hyperplane tessellations. Both basic limiting regimes are considered, the increasing domain asymptotics (expanding domain, fixed intensity) and the infill asymptotics (fixed domain, increasing intensity).

References


Crack detection in lithium-ion battery cells using machine learning

Lukas Petrich
lukas.petrich@uni-ulm.de

Abstract
It is an open question how the particle microstructure of a lithium ion electrode influences a potential thermal runaway. In order to investigate this, information on the structural changes, in particular cracked particles, caused by the failure are desirable. For a reliable analysis of these changes a reasonable large amount of data is necessary, which brings the need for automatic extraction of particle cracks from tomographic 3D image data. A classification model is proposed which is able to decide whether a pair of particles is the result of breakage, an image segmentation artefact or neither [2]. The classifier is developed using simulated data based on a 3D stochastic particle model [1]. Its validity is tested by applying the methodology to hand-labelled data from a real electrode. For this dataset an overall accuracy of 73% is achieved.

References

Statistical analysis of dependencies between different types of submissions to municipalities in the Czech Republic

Anna Pidnebesna

pidneann@fel.cvut.cz

Coauthors: Kateřina Helisová, Jakub Staněk

Abstract
Last years multiple time series analysis plays very important role in theory and application of wide set of science branches. Study of groups of submissions among municipalities in Czech Republic which can be described by multiple time series is presented. The main aim of this project was to describe the typical behavior of mentioned submissions, find the dependencies among their parts and build the suitable model. Modern approaches such as linear and correlation analysis and methods for detecting change points were used. During the analysis were observed change points and linear dependencies which can be meaningfully explained by existing laws and procedures of submissions of documents. Obtained results could be interested in economic, logistic and further work of municipalities.

References
Semiparametric estimation in planar segment processes

Milan Pultar\textsuperscript{2}
milan.pultar@matfyz.cz

Coauthors: Viktor Beneš, Jakub Večeřa (Charles University, Faculty of Mathematics and Physics, Department of Probability and Mathematical Statistics)

Abstract
We consider an inhomogeneous planar segment process with given probability density with respect to the Poisson segment process. From simulated realizations of the process using the maximum pseudolikelihood method we first estimate scalar parameters of the model. Then using the kernel density estimator of the Palm mark distribution we compute the estimate of the reference probability density on the set of all possible lengths of segments. We compare both the estimators and computed density with true parameters and reference density of the model. In our presentation we focus on the computational part of the problem.

Acknowledgment: The research was supported by the Czech Science Foundation, project 16-03708S.
Stochastic microstructure modeling of particle-based materials in 3D and 4D

Volker Schmidt

volker.schmidt@uni-ulm.de

Abstract

The microstructure of particle-based materials has a strong influence on their macroscopic physical properties, e.g., with respect to mass, charge and heat transport, mechanical strength, or degradation effects. Thus, optimizing the microstructure with respect to functionality is a main goal in materials research. Doing so experimentally in the laboratory causes high costs with regard to time and resources. One way to overcome this problem is the usage of stochastic microstructure models, which allow the realization of virtual microstructures on the computer. The functionality of microstructures generated with such models can be investigated by means of numerical simulations. The results of this combination of stochastic and numerical methods, which is called virtual materials testing, can help to design materials with improved microstructures that lead to better macroscopic functionalities.

In this talk we consider spatial stochastic models for the microstructure of two types of particle-based materials. First, we consider the internal 3D microstructure of single agglomerates and show that it has a great influence on their stability and breakage characteristics. To see this, spatial stochastic modeling of spherical agglomerates is combined with numerical DEM simulations for better understanding the breakage behavior of spherical agglomerates under uniaxial compression [1,2].

Second, we consider a (dynamic) stochastic simulation model for 3D grain morphologies undergoing a grain coarsening phenomenon known as Ostwald ripening. For this purpose, a compact and tractable representation of the grain structure is an extremely valuable tool. Tessellations have proven to be very good choices for such representations [3]. For low volume fractions of the coarsening phase, the classical LSW theory predicts a power-law evolution of the mean particle size and convergence toward self-similarity of the particle size distribution; experiments suggest that this behavior holds also for high-volume fractions. Using this we developed a stochastic simulation model for the 3D morphology of the coarsening grains at arbitrary time steps. Our stochastic model is based on random Laguerre tessellations and is by definition self-similar, i.e., it depends only on the mean particle diameter, which in turn can be estimated at each point in time [4]. Furthermore, a stochastic model is proposed that describes the statistical evolution of the "typical" individual grain size as a function of neighborhood characteristics [5]. This extends the existing 3D model to 4D.
References


Quermass-interaction process with convex compact grains - Ruelle stability

Jakub Staněk
stanekj@karlin.mff.cuni.cz

Coauthor: Kateřina Helisová

Abstract
In the last years, Quermass-interaction process has been studied because of its spread applications, for example in biology or medicine. In this contribution, we focus on Ruelle stability of the Quermass-interaction process with non circular grains in $\mathbb{R}^2$. Here, we present some corrections of condition for Ruelle stability introduced in [2] as well as new result for the process with grains having smooth boundary.

References


Abstract

The Brownian net is a continuum random object that arises as the diffusive scaling limit of one-dimensional systems of branching and coalescing particles. In the case without branching, it is called the Brownian web. The latter arose from the work of Arratia [1,2], was studied by Tóth and Werner [10], and further formalized by Fontes, Isopi, Newman, and Ravishankar [3]. The Brownian net (which includes branching) was introduced by Sun and Swart [7] and, by a different construction, by Newman, Ravishankar, and Schertzer [5], who also introduced the Brownian net with killing [6]. Further properties of the Brownian net were studied by Sun, Swart, and Schertzer [8,9]. Although the Brownian net is conjectured to be a universal scaling object, convergence for non nearest-neighbor random walks has so far only been proved in the case without branching [4]. In my talk, I will give a short introduction to the theory of the Brownian net.

References


Evolution equations with multiplicative fractional noise

Jana Šnupárková\textsuperscript{14}
Jana.Snuparkova@vscht.cz

Coauthor: Bohdan Maslowski\textsuperscript{2}

Abstract
In the talk, the evolution equations with multiplicative fractional noise will be discussed.
Data-driven selection of tessellation models
describing polycrystalline microstructures

Ondřej Šedivý

ondrej.sedivy@uni-ulm.de

Abstract
Tessellation models have been extensively used in literature for modelling grain microstructures of polycrystalline materials. Most of the studies have focused on one or a couple of models which are believed to describe the microstructure sufficiently well. We aim to provide a unified approach towards fitting a wide range of tessellation models to the particular dataset and suggest reasonable criteria for model selection. The portfolio of the models ranges from simple tessellations with convex grains to the most sophisticated models used so far for modelling polycrystalline microstructures. The latter models are based on an initial ellipsoidal approximation of grains [1].

In general, there are two contradictive factors influencing the model selection - accuracy and complexity of the model. Obviously, the selected model is supposed to be accurate enough to describe the microstructure sufficiently well. On the other hand, it should be as simple as possible and should not contain more parameters than needed to attain a prescribed accuracy. Thus, a trade-off between accuracy and complexity of the model needs to be found. While there is a number of measures of the accuracy in both topological and metrical sense, measuring the complexity is rather intuitive and mainly represented by the number of parameters in the model.

References

Stochastic microstructure modeling of battery anodes for virtual materials testing

Daniel Westhoff

daniel.westhoff@uni-ulm.de

Abstract
Lithium-ion batteries are an important field of current research. While in mobile devices, performance and durability of lithium-ion batteries is already quite satisfying, application in electric vehicles is still challenging. It is well known that the microstructure of the electrodes has an important impact on the functionality. An appropriate approach for improving battery performance is via stochastic modeling. This means, parametric stochastic 3D models are fitted to experimental data sets gained from 3D microscopy. Then, by changing model parameters, virtual but still realistic microstructures can be created on the computer, and their elect electrochemical performance can be analysed using spatially resolved transport models.

In order to do so, the solid phase of a battery anode is interpreted as a random closed set $\Xi \subset \mathbb{R}^3$. In this talk, a very flexible way for modeling this random closed set is presented, see [1,2,4]. We also show how an electrochemical simulation tool can be used to analyse the electrochemical behaviour of virtual anode microstructures on the computer. The result can be seen as a random marked closed set. This procedure is used for validation purposes of the stochastic microstructure model [3].

In addition, we account for the fact that the solid phase of a battery anode is not completely homogenous, i.e., there are local differences within the solid phase. This can be seen when considering the grayscale microscopy data. We show that random marked closed sets can be used to include this information.

References


Process of interacting line segment-time extension

Markéta Zikmundová
marketa.zikmundova@vscht.cz

Abstract
Consider the segment point process on the bounded set $S \subset \mathbb{R}^2$ given by the density

$$p(y|x) = c_x^{-1} \exp(x_1 \cdot N(U_y), x_2 \cdot L(U_y), x_3 \cdot I(U_y)),$$

w.r.t. Poisson segment process. Symbol $N(U_y)$ denotes the number of intersections of the segments in the union $U$ of configuration $y$. Similarly $L$ corresponds to the total length of the segments in $U$ and $I$ to the number of isolated segments. Vector $x = (x_1, x_2, x_3)$ is the vector of parameters of the density $p$. We want to estimate the parameter $x$ and set the space-time extension of the model.
List of institutions

1. Charles University, Faculty of Mathematics and Physics, Department of Mathematics Education, Sokolovská 83, 186 75 Prague 8, Czech Republic
2. Charles University, Faculty of Mathematics and Physics, Department of Probability and Mathematical Statistics, Sokolovská 83, 186 75 Prague 8, Czech Republic
3. Czech Technical University in Prague, Faculty of Electrical Engineering, Department of Economics, Management and Humanities, Technická 2, 166 27 Prague 6, Czech Republic
4. Czech Technical University in Prague, Faculty of Electrical Engineering, Department of Mathematics, Technická 2, 166 27 Prague 6, Czech Republic
5. Czech Technical University in Prague, Faculty of Transportation Sciences, Department of Applied Mathematics, Na Florenci 25, 110 00 Prague 1, Czech Republic
6. Friedrich-Schiller-Universität Jena, Institut für Mathematik, Ernst-Abbe-Platz 2, 07743 Jena, Germany
8. Technical University of Liberec, Department of Applied Mathematics, Studentská 1402/2, 461 17 Liberec 1, Czech Republic
9. The Czech Academy of Science, Institute of Computer Science, Pod Vodárenskou věží 2, 182 07 Prague 8, Czech Republic
10. The Czech Academy of Sciences, Institute of Information Theory and Automation, Pod Vodárenskou věží 4, 182 08 Prague 8, Czech Republic
11. The Czech Academy of Science, Institute of Physics, Na Slovance 1999/2, 182 21 Prague 8, Czech Republic
12. The Czech Academy of Science, Institute of Physiology, Vídeňská 1083, Prague 4, Czech Republic
13. The Czech Academy of Science, Institute of Thermomechanics, Dolejškova 1402/5, 182 00 Prague 8, Czech Republic
14. University of Chemistry and Technology Prague, Faculty of Chemical Engineering, Department of Mathematics, Technická 3, 166 28 Prague 6, Czech Republic
15. University of South Bohemia, Faculty of Economics, Department of Applied Mathematics and Computer Science, Studentská 13, 370 05 České Budějovice, Czech Republic
16. University of Split, Faculty of Science, Department of Mathematics, Teslina 12, 21000 Split, Croatia
17. University of Ulm, Faculty of Mathematics and Economics, Institute of Stochastics, Helmholtzstr. 18, 89069 Ulm, Germany